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Estimating the uncertainty of damage costs of pollution: A simple transparent method and typical results

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Abstract

Whereas the uncertainty of environmental impacts and damage costs is usually estimated by means of a Monte Carlo calculation, this paper shows that most (and in many cases all) of the uncertainty calculation involves products and/or sums of products and can be accomplished with an analytic solution which is simple and transparent. We present our own assessment of the component uncertainties and calculate the total uncertainty for the impacts and damage costs of the classical air pollutants; results for a Monte Carlo calculation for the dispersion part are also shown. The distribution of the damage costs is approximately lognormal and can be characterized in terms of geometric mean μ_g and geometric standard deviation σ_g, implying that the confidence interval is multiplicative. We find that for the classical air pollutants σ_g is approximately 3 and the 68% confidence interval is $[\mu_g/\sigma_g, \mu_g \sigma_g]$. Because the lognormal distribution is highly skewed for large σ_g, the median is significantly smaller than the mean. We also consider the case where several lognormally distributed damage costs are added, for example to obtain the total damage cost due to all the air pollutants emitted by a power plant, and we find that the relative error of the sum can be significantly smaller than the relative errors of the summands. Even though the distribution for such sums is not exactly lognormal, we present a simple lognormal approximation that is quite adequate for most applications.

Keywords: Monte Carlo calculation; Geometric standard deviation; Lognormal distribution; Dispersion models; Dose-response functions; Monetary valuation

1. Introduction

The uncertainties of environmental damages are far too large for the usual error analysis of physics and engineering (using only the first term in a Taylor expansion). Rigorous systematic assessment of the uncertainties is difficult and few studies have attempted it. Most merely indicate an upper and a lower value, but based on the range of just one input parameter or by simply combining the upper and lower bounds of several inputs, without taking into account the combination of uncertainties (e.g. of atmospheric dispersion, dose-response function and monetary valuation). Many damage assessments involve so many different inputs that an analytical solution was usually not considered, and of the uncertainty analyses that have been done, almost all use Monte Carlo techniques and numerical calculations (see e.g. Morgan et al., 1984, and Morgan and Henrion, 1990). The Monte Carlo method is powerful, capable of treating any problem, but it is computationally intensive and the result is "black box":

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it is difficult to see how important each of the component uncertainties is or how the result would change if a component uncertainty changes — especially for a reader who does not have access to the details of the calculations.

As a simple and transparent alternative the present paper presents an analytic method based on lognormal distributions. The justification lies in the observation that the calculation involves essentially a product of factors, and that the resulting uncertainty of the product is approximately lognormal for most damage costs of pollution. Thus it suffices to specify geometric mean and geometric standard deviations, or equivalently, multiplicative confidence intervals about the geometric mean (which is usually close to the median for damage costs but very different from the ordinary mean). Compared to a Monte Carlo analysis, this method yields typical answers that are easy to apply and communicate; the calculation is simple enough to allow the reader to modify the assumptions and see the consequences. Furthermore, the analytic method can be combined with Monte Carlo results for certain parts of the calculation, thus benefiting from the best features of each approach.

Whatever the method, an assessment of the uncertainties of damage costs must begin with a detailed examination of the uncertainties of each of the inputs to the impact pathway analysis, to estimate standard deviation and shape of the probability distribution of their uncertainties. This involves expert judgment with its unavoidably subjective aspects. These component uncertainties are then combined to obtain the total uncertainty of the damage cost. We describe the process and present results for mortality, the most important impact of the classical air pollutants (NO_x, PM, and SO_2), updating and refining earlier estimates of Rabl and Spadaro (1999).

The usefulness of lognormal distributions for the assessment of uncertainties in the environmental field has of course been recognized by many authors, for instance by Hammitt and Cave (1991), Slob (1994), and Burmaster and his colleagues in numerous publications (e.g. Burmaster and Hull, 1997, Burmaster, 1998, Burmaster and Crouch, 1997). The novel features of the present paper are: (i) a combination of the analytic method with a Monte Carlo analysis for the atmospheric modeling (a step of the impact calculation which seems too complicated for an analytic approach); (ii) a discussion of the placement of the confidence intervals with respect to the mean rather than the median (an issue that does not seem to have received sufficient attention); (iii) the results for the damage costs of NO_x, PM and SO_2; (iv) a simple and remarkably accurate method for estimating the geometric standard deviation of the sum of lognormally distributed variables (needed when damage costs for several different impacts or pollutants are added, the results being also approximately lognormal for cases of interest).

2. Methodologies for estimation of uncertainty

2.1. The calculation of damage costs

For pollutants that are harmful by inhalation the exposure-response function (ERF) or dose-response function is usually stated in terms of ambient concentration as concentration-response function (CRF). In terms of the slope s_{CR} of the CRF, the impact rate due to inhalation of an air pollutant can be written in the form

$$I(q) = \int dx \int dy \ \rho(\mathbf{x}) s_{CR}(\mathbf{x}) c_{air}(\mathbf{x}, q) \qquad (1)$$

where

$I(q)$	impact rate [cases/yr],
q	emission rate of pollutant [kg/yr],
$c_{air}(\mathbf{x},q)$	increase in concentration [$\mu g/m^3$] at a point $\mathbf{x}=(x,y)$ due to the emission q,
$\rho(\mathbf{x})$	density of receptors (population, buildings, crops,…) [receptors/m^2] at $\mathbf{x}$, and
$s_{CR}(\mathbf{x})$	slope of CRF at $\mathbf{x}$ [(cases/yr)/(receptor ($\mu g/m^3$))].

According to current knowledge the population-level CRFs for health impacts of the classical air pollutants (NO_x, PM, and SO_2) appear to be linear without threshold, and a single calculation is sufficient, using for $c_{air}(\mathbf{x},q)$ the annual average concentration. For nonlinear CRFs Eq. (1) should be used separately for different concentration ranges. If the CRF or ERF has the form of a hockey stick with a no-effect threshold, the integrand of Eq. (1) contains an additional factor $f_{thresh}(\mathbf{x})$, the fraction of the receptors at $\mathbf{x}$ that is above the threshold. As pointed out by Crawford and Wilson (1996), linearity is also the appropriate assumption for the calculation of small incremental changes — the most common situation in practical applications.

For some persistent pollutants, in particular As, Hg, Pb, and dioxins, some or most of the damage is due to ingestion, and the ingestion dose can be one to three orders of magnitude larger than the inhalation dose (Spadaro and Rabl, 2004). The calculation of impacts due to ingestion has the same form as Eq. (1), with obvious replacements: $s_{CR}(\mathbf{x})$ is replaced by the slope of the appropriate ERF, and if the ERF is based on the

concentration in food or water, $c_{air}(\mathbf{x},q)$ is replaced by the appropriate concentration. If the ERF is based on the ingested dose, an additional factor is needed to convert from concentration to dose.

The impact per emitted quantity is the ratio of I and q, designated here by the symbol D (for damage, in physical units)

$$D = I(q)/q, \tag{2}$$

and multiplication by the unit cost p [€/case] yields the damage cost C in €/kg for the impact in question. For many pollutants the concentration increase $c_{air}(\mathbf{x},q)$ is proportional to the incremental emission q and therefore D and the damage cost are independent of q. For some secondary pollutants, especially O_3 and nitrates, the relation can be nonlinear if q is large; however, for marginal impacts q is small and one can still assume linearity. The magnitude of q should be chosen as appropriate for the policy application(s) of interest.

The total damage cost of the pollutant is obtained by summing the individual C_i over all impacts i caused by this pollutant (for health the various impacts are called end points).

$$C = \sum_i D_i p_i. \tag{3}$$

Assessments by EPA (Abt, 2000, 2004) and by the ExternE project series [1998, 2000 and 2004] have found that more than 95% of the total quantified damage cost (for each air pollutant with the exception of O_3 and greenhouse gases) is due to health impacts. Since the CRF slopes s_{CR} for health impacts are assumed to be independent of $\mathbf{x}$, s_{CR} can be taken outside the integral. Let us designate the remaining integral, divided by the emission rate, as exposure E per emitted quantity of pollutant

$$E = \int dx \int dy \;\; \rho(\mathbf{x}) c_{air}(\mathbf{x},q)/q. \tag{4}$$

This is the contribution of inhalation to the intake fraction as defined by Bennett et al. (2002), i.e. the fraction of the emitted pollutant that passes through human bodies. Thus for health impacts (and any other impact whose s_{CR} is independent of $\mathbf{x}$) the damage cost can be written as

$$C = E \sum_i s_{CRi} \;\; p_i. \tag{5}$$

Since E involves the integration of a complicated function, its uncertainty is more difficult to evaluate, although one can simplify by using approximations, in particular replacing the integral by a sum over finite areas. Furthermore, as shown in the following section, there is a very simple approximation, the "uniform world model" (UWM) that yields results for typical situations (Section 3.1.1). With this model the uncertainty can be estimated with an explicit formula. A more detailed Monte Carlo calculation is described in Section 3.1.3.

For pollutants that are harmful by ingestion or dermal contact the calculations are more involved but the basic structure of the equations (combination of sums and products) is similar if transfer factors are used, as shown by Spadaro and Rabl (2004).

2.2. Uncertainty of sums and products

2.2.1. Sums

The UWM for the damage cost of a single impact involves a simple product. For many pollutants a single impact, mortality, contributes more than two thirds of the total damage cost (ORNL/RFF, 1994; Rowe et al., 1995; Abt, 2000, 2004; ExternE, 1998, 2000, 2004), and the uncertainty of the mortality cost can be taken as a good estimate for the sum of the impacts, as shown in Section 4. If several impacts make a significant contribution, one also has to sum over such products for the total damage cost. For sums and products an analytical solution is possible.

To begin, consider the sum

$$y = x_i + x_2 + \ldots + x_n \tag{6}$$

of uncorrelated random variables x_i. For example, using the UWM, the total cost due to the health impacts of PM can be written as the sum over endpoints i

$$C = (\rho/v_{dep}) \sum_i p_i s_{CR,i}$$

where the costs p_i and the CRF slopes $s_{CR,i}$ are uncorrelated between different impacts i (ρ is the receptor density and v_{dep} is a velocity that accounts for deposition and/or depletion of the pollutant).

The mean of y is

$$\mu_y = \mu_{x1} + \mu_{x2} + \ldots + \mu_{xn} \tag{7}$$

where the μ_{xi} are the means of the x_i. The standard deviation σ_y of y is given by the usual quadratic combination

$$\sigma_y^2 = \sigma_{x1}^2 + \sigma_{x2}^2 + \ldots + \sigma_{xn}^2 \tag{8}$$

of the standard deviations σ_{xi} of the x_i. Even though these relations are exact, regardless of the size of the standard deviations, they do not yield an interpretation of σ_y in terms of confidence intervals. For that one also needs the

probability distribution of y. Fortunately in many cases of practical interest the distributions are approximately Gaussian (also called normal). In particular, in the limit where the number of terms in the sum becomes large, the central limit theorem of statistics implies that the distribution of y approaches a Gaussian regardless of the individual distributions of the terms in the sum. In practice the distribution of y is close to a Gaussian unless one or several of the terms have distributions that have large standard deviations and are very different from Gaussian. When the distribution of y is nearly Gaussian one can say that $[\mu_y-\sigma_y, \mu_y+\sigma_y]$ is approximately the 68% confidence interval and $[\mu_y-2\sigma_y, \mu_y+2\sigma_y]$ approximately the 95% confidence interval.

2.2.2. Products

These considerations apply also to the product z of uncorrelated variables x_i

$$z = x_1 x_2 x_3 \ldots x_n \tag{9}$$

if one looks at the logarithm. For example, the factors of the UWM are uncorrelated with each other. The mean of the logarithm of a random variable is the logarithm of the geometric mean μ_g; specifically, if $p(z)$ is the probability distribution of z, the geometric mean is given by

$$\ln(\mu_{gz}) = \int_0^\infty p(z)\ln(z)\mathrm{d}z \tag{10}$$

Since the mean of $\ln(\mu_{gz})$ is the sum of the logarithms of the geometric means μ_{gxi} of the x_i, μ_{gz} is given by the product

$$\mu_{gz} = \mu_{gx1}\mu_{gx2}\ldots\mu_{gxn}. \tag{11}$$

Let us now define the geometric standard deviation σ_{gz} as

$$[\ln(\sigma_{gz})]^2 = \int_0^\infty p(z)[\ln(z) - \ln(\mu_{gz})]^2\mathrm{d}z \tag{12}$$

and analogously for the x_i. Assuming independence of the distributions one finds that the geometric standard deviation σ_{gz} of the product z is given by

$$[\ln(\sigma_{gz})]^2 = [\ln(\sigma_{gx1})]^2 + [\ln(\sigma_{gx2})]^2 + \ldots + [\ln(\sigma_{gxn})]^2. \tag{13}$$

2.3. The lognormal distribution

In practice the lognormal distribution is far more common than most people realize, a situation highlighted in an interesting review by Limpert et al. (2001). The lognormal distribution of a variable z is obtained by assuming that the logarithm of z has a normal distribution (Morgan and Henrion, 1990). Invoking the central limit theorem for the product z, one sees that the lognormal distribution is the "natural" distribution for multiplicative processes, the same way that the Gaussian distribution is natural for additive processes. Although the lognormal distribution becomes rigorous only in the limit of infinitely many factors, in practice it can be a good approximation even for a few factors, provided the distributions with the largest spread are not too far from lognormal. For many environmental impacts the lognormal model for the result is quite relevant because the impact is a product of factors and the distributions of the individual factors are not too far from lognormality. All one has to do is estimate the geometric standard deviations of the individual factors and combine them according to Eq. (13).

The lognormal probability density distribution is given by

$$p(z) = \frac{1}{\phi z\sqrt{2\pi}}\exp\left[-\frac{(\ln(z)-\xi)^2}{2\phi^2}\right], \tag{14}$$

where geometric mean μ_g and geometric standard deviation σ_g are related to ξ and ϕ by

$$\mu_g = \exp(\xi) \text{ and } \sigma_g = \exp(\phi). \tag{15}$$

An example is shown in Fig. 1. For the lognormal distribution the geometric mean μ_g is equal to the median. If a quantity with a lognormal distribution has a geometric mean μ_g and a geometric standard deviation σ_g, the probability is approximately 68% for the true value to be in the interval $[\mu_g/\sigma_g, \mu_g\,\sigma_g,]$ and 95% for it to be in the interval $[\mu_g/\sigma_g^2, \mu_g\,\sigma_g^2]$.

One can show that the ordinary mean μ and standard deviation σ of the lognormal variable z are given by

$$\mu = \exp(\xi + \phi^2/2) = \mu_g\exp\left(\frac{[\ln(\sigma_g)]^2}{2}\right) \tag{16}$$

and

$$\sigma = \sqrt{[\exp(\phi^2)-1]\exp(2\xi+\phi^2)} = \mu\sqrt{(\mu/\mu_g)^2 - 1}. \tag{17}$$

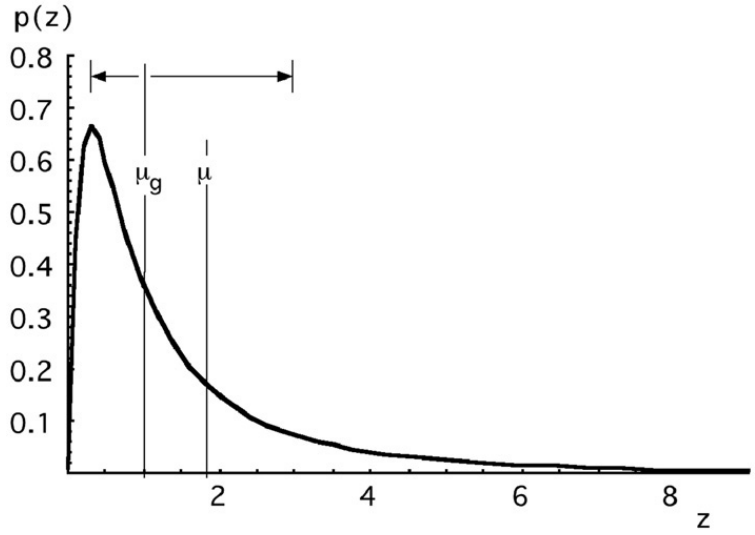

Fig. 1. Lognormal probability density function ($\mu_g=1$, $\sigma_g=3$, mean $\mu=1.83$). Arrows indicate the 68% confidence interval.

Given any one of the pairs $\{\xi, \phi\}$, $\{\mu, \sigma\}$ or $\{\mu_g, \sigma_g\}$, the others can be determined by means of Eqs. (16) and (17).

3. Component uncertainties and results for air pollution

3.1. Models

For air pollution by far the most complex part of the analysis concerns the dispersion and chemistry of the pollutant in the atmosphere. Many models are available, ranging from simple Gaussian plume models to very detailed and computationally intensive Eulerian models. In this section we review those aspects that affect the uncertainty. We begin with the "uniform world model" because it provides a convenient and instructive shortcut that yields approximate results. That is followed with a more general discussion of atmospheric models.

3.1.1. The "Uniform World Model" (UWM)

Most policy applications concern pollution sources the sites of which are not known in advance, and therefore one needs typical damage estimates rather than numbers for a specific installation. A simple and convenient tool for the estimation of typical values of the exposure E of Eq. (4) is the UWM, first presented by Curtiss and Rabl (1996) and further developed, with detailed validation studies, by Spadaro (1999) and Spadaro and Rabl (1999, 2002). More recently Spadaro and Rabl (2004) have extended the UWM to toxic metals and their pathways through the food chain. The UWM is a product of a few factors; it is simple and transparent, showing at a glance the role of the most important parameters of the impact pathway analysis. It is exact (because of the conservation of matter) in the limit where the distribution of either the sources or the receptors is uniform and the key atmospheric parameters are the same everywhere. In practice the agreement with detailed models is usually within a factor of two for primary pollutants and stack heights above 50 m. For secondary pollutants such as nitrates and sulfates the variation with stack height is negligible, and the agreement between UWM and detailed models is better than a factor of two.

The UWM for the damage cost D_{uni} of a particular impact due to the inhalation of a primary pollutant is shown in Eq. (18)

$$D_{uni} = p\, s_{CR}\, \rho / v_{dep} \tag{18}$$

where

p	cost per case ("price") [€/case],
s_{CR}	CRF slope [(cases/yr)/(pers(μg/m^3))],
ρ	average population density [pers/km^2] within 1000 km of source, and
v_{dep}	deposition velocity of pollutant (dry + wet) [m/s].

D_{uni} is the damage cost due to the emission of a specified quantity of the pollutant and has units of €/kg (the units indicated here are customary for the respective quantities, but in the equation they must of course be

converted to a consistent set). For secondary pollutants the equation has the same form, but v_{dep} is interpreted as depletion velocity, a quantity that includes the transformation rate of the primary into the secondary pollutant as well as the deposition velocities of the primary and secondary pollutants (Curtiss and Rabl, 1996). With this model it is easy to transfer to the results from one region to another (assuming that CRF and deposition velocity do not change): simply rescale the result in proportion to the receptor density and the cost per case.

3.1.2. Atmospheric models

For the uncertainty of atmospheric models a geometric standard deviation in the range from two to five is sometimes cited, but without making a distinction between episodic values and averages over space or time. In fact, atmospheric models are far more accurate for averages than for episodic values. This is an important consideration since for policy applications, of interest here, one needs long term average values rather than episodic values. For example, the European tracer experiment (ETEX) (van Dop et al., 1998) has provided validation for a variety of dispersion models, but on an episodic basis; the relatively large discrepancies between measured and calculated values are therefore no indication of the accuracy that can be expected for long term averages. Generally models for nonreactive species have greater accuracy than models that include chemical reactions; the reactions of nitrogen compounds are especially difficult to model.

To compare long term average concentrations between models and measured data one needs either models at the continental scale, with a complete emissions inventory, or else special situations where a single source dominates the concentrations in a local zone (another possibility is to emit a tracer gas, but the quantity tends to be unacceptably large for long term tests). Comparisons for long term averages at the regional scale can be found in the reports of the EMEP Program (see for instance Barrett, 1992). They generally indicate agreement within a factor of two or better. At the local scale we found an interesting confirmation of the Gaussian plume model ISC (Brode and Wang, 1992) with SO_2 data for a refinery in Donges, near Nantes, France. The refinery is by far the dominant source of SO_2 in the region. The ISC results, with detailed emissions for the year 2000 including stack heights of all the sources at the refinery, indicate that the refinery contributes about 3.5 $\mu g/m^3$ on average at the measuring station some 2 km from the source. The annual averages measured in 2000 were 8.8 $\mu g/m^3$ at this station, and 3.6 $\mu g/m^3$ and 4.7 $\mu g/m^3$, respectively, in the nearby cities St.-Nazaire (about 15 km upwind) in Nantes (about 25 km downwind); the later two values can be taken as approximation of the background. The difference between the measured value 8.8 $\mu g/m^3$ near the refinery and the background of approximately 4.1 $\mu g/m^3$, i.e. 4.7 $\mu g/m^3$, agrees reasonably well with the calculated contribution of 3.5 $\mu g/m^3$ from the refinery.

Among the many parameters and input data of an atmospheric model, most have only a relatively minor effect on the calculation of long term average concentrations. To see which parameters are the most important, it is instructive to look at the "uniform world model" (UWM) of Section 3.1.1, because it yields the damage costs for typical conditions. In the UWM the key parameters of a dispersion model are those that affect the deposition and/or depletion velocity v_{dep}. For an indication of the kind of distribution that can be expected, we show in Fig. 2 a histogram of dry deposition velocity data for SO_2, based on a review by Sehmel (1980). Visibly, a logarithmic scale is much more appropriate for these data than a linear one. The geometric standard deviation is approximately 2.5 for this sample. The variability of this sample is due to different surface materials, atmospheric conditions and variation with time of day and year. More recent data may have a smaller standard deviation, but we have not been able to find a survey as comprehensive as that of Sehmel. Even though variability does not imply uncertainties if the model accounts correctly for all of its causes, in practice most models cannot treat all the necessary detail and so the variability increases the uncertainties.

For particles we refer to Fig. 19.3 of Seinfeld and Pandis (1998), which likewise suggests a lognormal distribution. Dry deposition of small particles has been reviewed by Nicholson (1988), who points out the large variability of measured deposition velocities with the nature of the surface and the conditions of the observation. The spread of values seems to be comparable to Fig. 2 for SO_2.

Deposition velocities for reactive nitrogen compounds have been reviewed by Hanson and Lindberg (1991); here the variability with the conditions of the absorbing surface is further enhanced by the high chemical reactivity of nitrogen compounds.

The possibility of low values of dry deposition velocities could imply very large damages under dry conditions. However, for the wet climates typical of Europe, long distance dispersion will be limited by wet deposition. We have verified this for particulate matter by varying the dry deposition velocity in the ECOSENSE

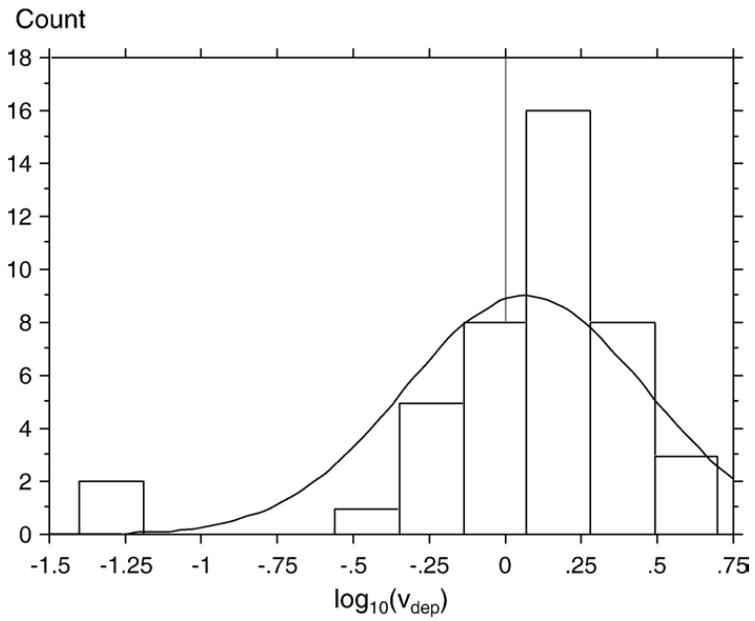

Fig. 2. Distribution and lognormal fit of maximum values, in the review of Sehmel (1980), for dry deposition velocity [in cm/s] of SO_2 over different surfaces.

model (Krewitt et al., 1995), the model used by ExternE. Thus the uncertainty of total deposition in Europe appears to be significantly smaller than suggested by dry deposition data.

3.1.3. A Monte Carlo analysis of exposure

By contrast to the typical results of the UWM, for site-specific calculations the exposure involves the integration over the entire region of the product of receptor density and concentrations, the latter calculated by detailed dispersion models. We have estimated the uncertainty of exposure by means of a Monte Carlo calculation, taking into account the uncertainties of the numerous input data (Spadaro and Rabl, 2005). Probability distributions are used for the possible values of the input parameters. Since some of the distributions are not well known, several possible cases are considered. Only dispersion is taken into account, without chemical reactions.

The analysis for primary air pollutants starts from the mass balance for the average pollutant concentration in a column of air that moves with the wind from source to receptors. For an initial analysis we have made several assumptions to calculate the ground level concentrations; they are generally made by models that calculate collective exposure and are believed to be acceptable for that purpose:

- A1 the pollutant moves along straight trajectories away from the source;
- A2 the wind speed does not vary with height (although for a given stack height we take this wind speed to be the value calculated at the stack height via a power–law relationship);
- A3 wind speed, mixing layer height and atmospheric stability class are constant for a puff moving along its trajectory;
- A4 there is no exchange with the upper atmosphere above the mixing layer height;
- A5 the distributions of the parameters are statistically independent;
- A6 for the ratio of the ground level and the column-average concentrations one can take the ratio of a Gaussian plume dispersion model, multiplied by a random number with a lognormal distribution (this is to take into account the fact that the real concentrations are not vertically uniform).

Then a sensitivity analysis is carried out to show that the results do not change significantly when these assumptions are relaxed. This approach provides a model-independent assessment of the uncertainty of any dispersion model that satisfies the assumptions, including the model used by the ECOSENSE (Krewitt et al., 1995) software of the ExtenE project series.

Results for the dispersion of primary pollutants have been obtained with stack height 75 m and typical plume rise at three locations: a very large population center (Paris), an intermediate site (Lauffen near Stuttgart), and a rural site (Albi in the Southwest of France). The

uncertainty of the collective exposure, expressed as geometric standard deviation σ_g, ranges from about 1.2 for Paris and 1.5 for Stuttgart to about 1.9 for Albi. The uncertainty is larger for rural sites because for a rural site the regional impacts dominate and the regional impacts are very sensitive to the assumptions about the deposition velocity, whereas deposition is almost negligible in the local zone (for PM, SO_2 and NO_x). Since most pollution sources tend to be located more in or around cities than in rural areas, we assume a σ_g of 1.5 for the dispersion modeling of primary non-reactive air pollutants. For the dispersion of secondary pollutants we take a larger σ_g of 1.7 because their formation takes place over large distances and the impacts are almost entirely regional, a situation more akin to Albi than Stuttgart.

For the dispersion of NO_x and SO_2 we take a somewhat larger σ_g because their impacts, being due to their secondary pollutants, occur mostly at greater distances from the source, thus rendering their dispersion aspects more like the rural situation for PM. These numbers are consistent with estimates by McKone and Ryan (1989). For secondary pollutants there is additional uncertainty due to chemistry, especially in the case of ozone, but the uncertainty due to inaccuracies in the spatial distribution of concentration values relative to receptors is much smaller because secondary pollutants form only gradually at distances removed from the source. Since the chemical reactions depend on the background concentrations which are not sufficiently well known, we also introduce a σ_g for the effect of background emissions.

These considerations lead us to assume

σ_g	1.5 for the dispersion of non-reactive primary pollutants
σ_g	1.7 for the dispersion of SO_2 and sulfates
σ_g	1.2 for the formation of sulfates from SO_2
σ_g	1.05 for the effect of background emissions on the formation of sulfates from SO_2
σ_g	1.7 for the dispersion of NO_x and nitrates
σ_g	1.4 for the formation of nitrates from NO_x
σ_g	1.15 for the effect of background emissions on the formation of nitrates from NO_x

The key parameters for these processes enter in approximately multiplicative fashion (in the UWM their combination is exactly multiplicative).

3.2. Exposure-response functions

The uncertainty of dose-response functions varies widely from case to case. Best established are the ones for health impacts from radionuclides, the ERFs for certain health impacts from the classical pollutants (PM_{10}, SO_2, NO_2 and O_3), and the ones for impacts of SO_2, NO_2 and O_3 on certain crops whose economic importance has prompted laboratory studies.

The confidence intervals of ERFs for health impacts are usually reported for 95% probability, and they are approximately symmetric (of the form $\mu \pm 2\sigma$) around the mean μ. The underlying probability distributions (implicit in the regression software used in the respective studies) are usually not lognormal, hence it is necessary to estimate the corresponding geometric standard deviations σ_g.

If one knows the probability distribution of the residuals in the respective studies, one could calculate the geometric standard deviation exactly from its definition in Eq. (12). If one does not, but the reported confidence intervals are symmetric, it is reasonable to assume a Gaussian distribution. Strictly speaking the resulting σ_g is complex because the Gaussian is nonzero at negative values. However, negative values are not plausible on physical grounds (for health impacts of air pollutants a beneficial effect is not plausible), and the distribution should be cut off at zero. Furthermore, if one uses only ERFs that are statistically significant at the 95% level, the contribution of the negative values represents at most 2.5% of the normalization integral of the Gaussian, and the effect on the resulting σ_g would be negligible.

A much simpler alternative is the following approximation. Suppose that $\mu \pm \sigma$ corresponds to a 68% confidence interval, as for a Gaussian distribution. Then one fits a corresponding lognormal distribution such that its 68% confidence interval equals $[\mu-\sigma, \mu+\sigma]$, which yields σ_g as

$$\sigma_g = \sqrt{\frac{\mu+\sigma}{\mu-\sigma}}. \tag{19}$$

We have evaluated Eq. (19) for all the ERFs of ExternE for NO_x, SO_2, PM and O_3; typically σ_g is in the range 1.2 to 1.8. Specifically for chronic mortality Table 2 of Pope et al. (2002) indicates, for the average exposure during the observation period, a relative risk RR given by $RR-1=0.06$ with 95% confidence interval [0.02, 0.11]. With $\mu=0.06$ and $\sigma=0.023$ we find $\sigma_g=1.48$. In the following we will take 1.5 as a typical value.

For chronic mortality one also needs to determine the relation between the YOLL (years of life lost) and the change in the age-specific mortality rate that has been reported by studies of chronic mortality. Leksell and Rabl (2001) have examined the uncertainties of this

calculation; their results suggest a σ_g of 1.3 for the calculation of the YOLL, given the relative risk.

There is, however, another type of uncertainty due to the difference between the PM in ambient air on which epidemiology is based and the primary and secondary PM in the damage calculations. Ambient PM is a mix of primary PM from combustion and secondary PM, especially nitrates (due to NO_x emissions) and sulfates (due to SO_2 emissions). For the damage calculations one needs assumptions about the relative toxicity of the different components of ambient PM. The uncertainty of these assumptions is difficult to estimate, see e.g. a recent workshop on this problem (COST, 2006). To deal with this issue we introduce a factor for the respective toxicities of primary particles, nitrates and sulfates relative to ambient PM, and we assume a σ_g for these toxicities (our choices are a subjective judgment based on extensive discussions with epidemiologists and toxicologists). There could also be important direct impacts of NO_2 and SO_2 but currently the dominant thinking among epidemiologists is that they are negligible compared to those of PM and O_3 (see e.g. WHO, 2003).

To sum up, we assume

σ_g 1.5 (range 1.2 to 1.8) for the morbidity ERFs due to ambient PM

σ_g 1.5 for the mortality risk (RR-1) due to ambient PM

σ_g 1.5 for the toxicity of primary particles relative to ambient PM

σ_g 2 for the toxicity of nitrates and sulfates relative to ambient PM

σ_g 1.3 for the calculation of the YOLL for a given mortality risk.

These elements enter the calculation in multiplicative fashion.

3.3. Monetary valuation

Some physical impacts can be easily valued by their price on the market, e.g. the price of crops. There is little uncertainty of these prices as quoted at any particular place and time; uncertainty comes mainly from their future evolution (since the interesting policy applications concern the future) and from possible errors in collecting the information. Geometric standard deviations around 1.1 to 1.3 seem reasonable.

Nonmarket goods are difficult to value economically. This is especially true for the reference value for the protection of human lives, often called *value of statistical life* although *value of a prevented fatality* (VPF) is more appropriate because it expresses the willingness to pay (WTP) to avoid a premature death rather than what people might feel is the intrinsic value of life. It is a difficult good to monetize, and the uncertainty is large. The distribution of VPF results from

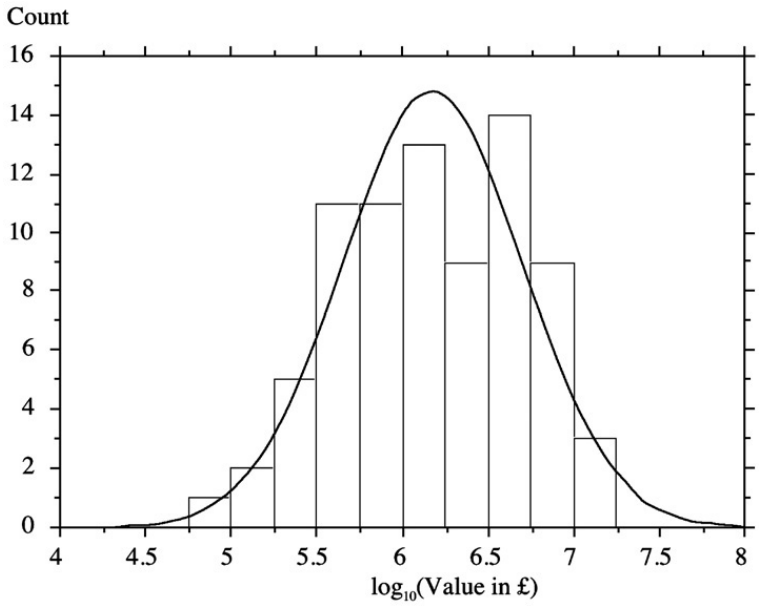

Fig. 3. Example of lognormal distribution for economic valuation: value of a prevented fatality (VPF), in £1990, as determined by 78 studies reviewed by Ives et al. (1993), histogram and lognormal fit plotted on log scale.

Table 1
Uncertainty of VPF: distributional characteristics from the survey by Ives et al. (1993)

Mean	Million £ 2.76 ($£_{1990}$ 1=$ 1.78)
Standard deviation	Million £ 3.00
Median	Million £ 1.59
Geometric mean μ_g	Million £ 1.49
Geometric standard deviation σ_g	3.4

various studies of individual preferences tends to be lognormal, as illustrated for example by Fig. 3 which is based on the Ives et al. (1993) survey of 78 VPF studies published between 1973 and 1990. Fig. 3 gives equal weight to all studies, regardless of quality or age; the resulting large spread of values could probably be reduced by applying reasonable selection criteria based on the benefit of hindsight.

The distribution is asymmetrical, and it has a large tail of high outliers, a situation typical of such valuation studies. As a consequence, if the spread is large, the mean is much larger than the median. This is illustrated in Table 1 which summarizes the distributional characteristics of Ives, Kemp and Thieme. The spread is so large that an interval of plus/minus one ordinary standard deviation extends to negative values. Clearly it does not make much sense to use ordinary mean and standard deviation in such cases. The median is far less affected by outliers, and in Table 1 it is fairly close to the geometric mean; the spread is best expressed by the geometric standard deviation.

There appears to be an emerging political consensus in Europe and North America that a value for VPF in the range of 1 to 5 M€ is reasonable. For example, the Department of Transport in the USA assumes $3 million and EPA $6 million; the DG Environment of the European Commission assumes values in the range of 1 to 1.5 M€ for air pollution mortality. Thus we will assume a σ_g of 2 for the value of life VPF. We also take σ_g of 2 for the value of a life year (VOLY) in view of the results of the VOLY studies carried out in the NewExt (2001–2003) and NEEDS (2004–2008) phases of ExternE. Since the unit cost of chronic bronchitis is based on contingent valuation, just like VOLY, we assume the same σ_g.

To sum up, we assume

σ_g 1.1 to 1.3 for market prices (cost of medical treatment, crop losses, repair cost for materials, etc.)

σ_g 2 for the WTP to avoid suffering or death, in particular for chronic bronchitis and for mortality (VPF and VOLY).

For morbidity endpoints other than chronic bronchitis the uncertainty lies between 1.1 and 2, depending on the extent to which the WTP to avoid the suffering dominates the medical expenditures. For restricted activity days we estimate a σ_g around 1.5.

3.4. Total σ_g of damage costs

Table 2 shows the assumptions for the component uncertainties and the result for the damage cost for mortality. Of course such choices involve expert judgment with their inevitable subjective aspects. However, it is easy for the reader to make different

Table 2
Uncertainty of damage cost estimates per kg of pollutant for mortality

	Lognormal?	σ_{gi} PM	$\ln(\sigma_{gi})^2$	σ_{gi} SO_2 via sulfates	$\ln(\sigma_{gi})^2$	σ_{gi} NO_x via nitrates	$\ln(\sigma_{gi})^2$
Exposure calculation							
Dispersion	Yes	1.5	0.164	1.7	0.282	1.7	0.282
Chemical transformation	Yes	1	0.000	1.2	0.033	1.4	0.113
Background emissions	No	1	0.000	1.05	0.002	1.15	0.020
Total σ_g for exposure		1.50	0.16	1.76	0.32	1.90	0.41
ERF							
Relative risk	No	1.5	0.164	1.5	0.164	1.5	0.164
Toxicity of PM components	?	1.5	0.164	2	0.480	2	0.480
YOLL, given relative risk	No?	1.3	0.069	1.3	0.069	1.3	0.069
Total σ_g for ERF		1.88	0.40	2.33	0.71	2.33	0.71
Monetary valuation							
Value of YOLL (VOLY)	Yes	2	0.480	2	0.480	2	0.480
Total (Eq. (13))		2.78	1.04	3.42	1.51	3.55	1.61

Sample calculations of geometric standard deviation σ_g, inserting the component uncertainties σ_{gi} into Eq. (13). The relative contributions of the σ_{gi} to total can be seen under $\ln(\sigma_{gi})^2$.

choices and calculate the corresponding σ_g. For sulfates and nitrates ExternE assumes the same ERFs as for PM (apart from an overall scale factor), therefore the contributions to the uncertainty are the same for each of these pollutants, with the exception of

i) atmospheric dispersion and chemistry (we assume different geometric standard deviations for PM, NO_x and SO_2).
ii) the toxicities of primary PM, sulfates and nitrates relative to ambient PM10, as discussed at the end of Section 3.2.

The resulting geometric standard deviations are 2.78 for primary PM, 3.26 for SO_2 and 3.39 for NO_x. The distribution of a product is exactly lognormal if each of the factors is lognormal. In practice it is sufficient for the factors with the largest widths to be approximately lognormal, a condition satisfied in the present case. Thus lognormality for the distribution of the result is very plausible for the damage costs.

We show three significant figures only to bring out the differences between these pollutants and the larger uncertainties of the secondary pollutants. But in view of the subjective and rather uncertain assumptions we had to make about the component uncertainties, we believe that it is best to simply sum up the results by saying that the geometric standard deviation of these damage costs is approximately 3. For chronic bronchitis the results are similar. For pollutants such as dioxins, As, Hg and Pb whose impacts come mostly from ingestion, we estimate, very roughly, that the geometric standard deviation is around 4.

3.5. Placement of the confidence intervals

A comment is required about the placement of the confidence intervals relative to the damage cost estimates. In fact, one needs to consider whether the key parameters of the calculations have been estimated as means, medians or something else, for instance modes (=point where the probability distribution of possible parameter values has its maximum). Our informal survey of the practice of researchers in the respective disciplines leads us to the conclusion that the typical choice is the mean. Contingent valuation studies might appear to be an exception to this rule because they frequently state the median rather than the mean willingness-to-pay; however, this is done to reduce the influence of extremely high responses that are clearly

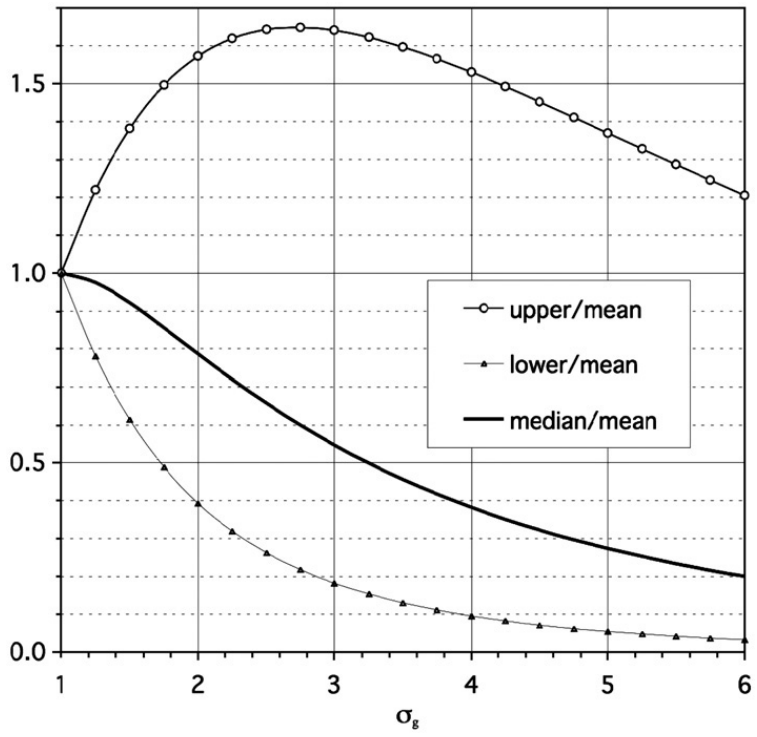

Fig. 4. Median (μ_g), upper bound ($\mu_g \times \sigma_g$) and lower bound (μ_g/σ_g), all divided by the mean μ, as function of the geometric standard deviation σ_g for a lognormal distribution.

unrealistic and would bias the result upward. The usual goal is to obtain the best estimate of the true population mean, as explained by Baker et al. (2006).

Since the confidence intervals that have been estimated for the damage costs are symmetric around the median (=geometric mean for lognormal distribution) on a logarithmic scale, their placement relative to the quoted damage costs has to be modified. Recalling Eq. (16) we note that for a lognormal distribution the ratio of mean μ and median μ_g is given by

$$\mu/\mu_g = \exp\left(\frac{[\ln(\sigma_g)]^2}{2}\right). \quad (20)$$

There is a sizeable difference between median and mean, as can be seen in Fig. 4 where the ratios median/mean, upper/mean and lower/mean are plotted as function of the geometric standard deviation σ_g for a lognormal distribution. For example, with $\sigma_g=3$ the mean/median ratio μ/μ_g is 1.83.

To see what this implies for the placement of the upper and lower bounds, let us consider the numbers for PM damage cost according to the UWM of Eq. (18). Table 3 shows the results for the placement of the confidence intervals. We take σ_g of the depletion velocity v_{dep} as the total for atmospheric modeling, namely the combination of the $\sigma_{g,i}$ for dispersion and chemical transformation, as per Table 2. For PM the ratio lower bound/mean is 0.59/2.78=0.21 and the upper bound/mean is 0.59*2.78=1.65.

Since the estimation of uncertainties is extremely uncertain, we believe that it would be appropriate to cite just a single set of results for air pollutants such as PM, SO_2 and NO_x that act via inhalation. For policy applications typical uncertainties are more instructive than detailed values of geometric standard deviation for each source and each impact. For inhalation of most air pollutants we suggest a typical geometric standard deviation of 3 and the ratios median/mean ≈ 0.5, low/mean ≈ 0.2 and high/mean ≈ 1.6. For toxic metals we estimate, very roughly, that σ_g might be around 4; the corresponding ratios are median/mean ≈ 0.38, low/mean ≈ 0.1 and high/mean ≈ 1.5.

Table 3
Placement of the 68% confidence intervals [Low, High] for UWM

	PM	SO_2 via sulfates	NO_x via nitrates
μ/μ_g for exposure (v_{dep})	1.09	1.17	1.23
μ/μ_g for CRF	1.22	1.43	1.43
μ/μ_g for monetary value p	1.27	1.27	1.27
μ/μ_g for UWM, Eq. (21)	1.68	2.13	2.23
μ_g/μ for UWM	0.59	0.47	0.45
Low/μ_g=1/σ_g for UWM	0.36	0.29	0.28
High/μ_g=σ_g for UWM	2.78	3.42	3.55
Low/μ for UWM	0.21	0.14	0.13
High/μ for UWM	1.65	1.61	1.59

Mean=μ, median=μ_g.

We note however, that the mean of the damage cost of the UWM cannot be calculated by simply plugging the mean of the depletion velocity v_{dep} (which determines the exposure) into Eq. (18). That is a consequence of the fact that the lognormally distributed v_{dep} is in the denominator rather than the numerator of the UWM of Eq. (18). The ratios μ/μ_g are given by Eq. (20), both for the product and for each factor, and $\ln(\sigma_g)$ squared of the product is given by the sum of the logarithms of the $\sigma_{g,j}$ of the individual factors j squared according to Eq. (13). Thus one readily obtains

$$\left(\frac{\mu}{\mu_g}\right)_{UWM} = \left(\frac{\mu}{\mu_g}\right)_{CR}\left(\frac{\mu}{\mu_g}\right)_{p}\left(\frac{\mu}{\mu_g}\right)_{\rho}\left(\frac{\mu}{\mu_g}\right)_{v_{dep}}. \quad (21)$$

(In practice one can set $\mu/\mu_g=1$ for ρ since the uncertainty of the receptor density ρ is negligible). According to Eq. (11) the geometric mean $\mu_{g,UWM}$ of the UWM is given by

$$\mu_{g,UWM} = \frac{\mu_{g,CR}\mu_{g,p}\mu_{g,\rho}}{\mu_{g,vdep}}. \quad (22)$$

For the geometric mean of a product it does not matter whether the factors are in the numerator or denominator. However, when one combines Eqs. (21) and (22) one finds that this is not true for the ordinary mean

$$\mu_{UWM} = \frac{\mu_{CR}\mu_p\mu_\rho}{\mu_{vdep}}\left(\frac{\mu_{vdep}}{\mu_{g,vdep}}\right)^2. \quad (23)$$

Therefore Eq. (18) of the UWM is correct for the geometric means but not for the ordinary means; the ordinary mean of the UWM has to be calculated according to Eq. (20). More generally, if a product contains terms in the denominator that have a lognormal distribution, the mean of the product is obtained from the product of the means via correction factors $(\mu_j/\mu_{g,j})^2$ for each such term j.

4. Sum over endpoints, impact categories or pollutants

According to Eq. (5) the damage costs of air pollutants involve sums of products (over endpoints, impact types or pollutants) and for that the analysis becomes more complicated, even if the terms of the sum

Table 4
Examples of combination of errors in a sum $w=v_1+v_2$ of two random variables v_1 and v_2, each with mean μ_i and relative error $\sigma_i/\mu_i=300\%$

	μ	σ	σ^2	Relative error (%)
a) First term is 80% of total				
v_1	0.8	2.4	5.76	300
v_2	0.2	0.6	0.36	300
$w=v_1+v_2$	1.0	2.47	6.12	247
b) First term is 65% of total				
v_1	0.65	1.95	3.80	300
v_2	0.35	1.05	1.10	300
$w=v_1+v_2$	1.0	2.21	4.91	221

are statistically independent as we assume here. Whereas it is easy to combine the geometric standard deviations of s_{CRi} and p_i to get that of the product $s_{CRi}\,p_i$, for the sum of such terms one needs the ordinary standard deviations of each product $s_{CRi}\,p_i$, according to Eq. (8). They can be obtained by means of Eqs. (16) and (17). Finally, having determined the ordinary standard deviation of the sum, one can use these equations again to find the corresponding geometric standard deviation. However, this latter step is not rigorous because Eqs. (16) and (17) are exact only for a lognormal distribution. Thus the analytical solution is more complicated and only approximate, and with so many parameters (for each of the endpoints) that the result is no longer very transparent.

Fortunately, in practice the calculations can often be greatly simplified by noting that thanks to the quadratic combination of errors small terms in the quadratic sum can be neglected. It is instructive to illustrate this phenomenon with the examples in Table 4 for the sum w of two random variables v_1 and v_2. The magnitude of their means μ_1 and μ_2 in part b) is chosen to correspond roughly to the relative contributions of mortality and the other impacts to the total damage cost of PM, NO_x and SO_2 according to ExternE (2004). If both terms have the same relative error σ_i/μ_i (here taken as 300%), the contribution of the error of the smaller term to the total error of the sum is quite small, as shown by the column under σ^2.

Furthermore, if a single term dominates, as is the case of mortality for the damage cost of the classical air pollutants, the error of the sum is not very different from that of the largest summand. Even in part b) the difference between the relative errors of the larger term (300%) and of the sum (221%) does not appear very significant in view of the subjectivity of any uncertainty

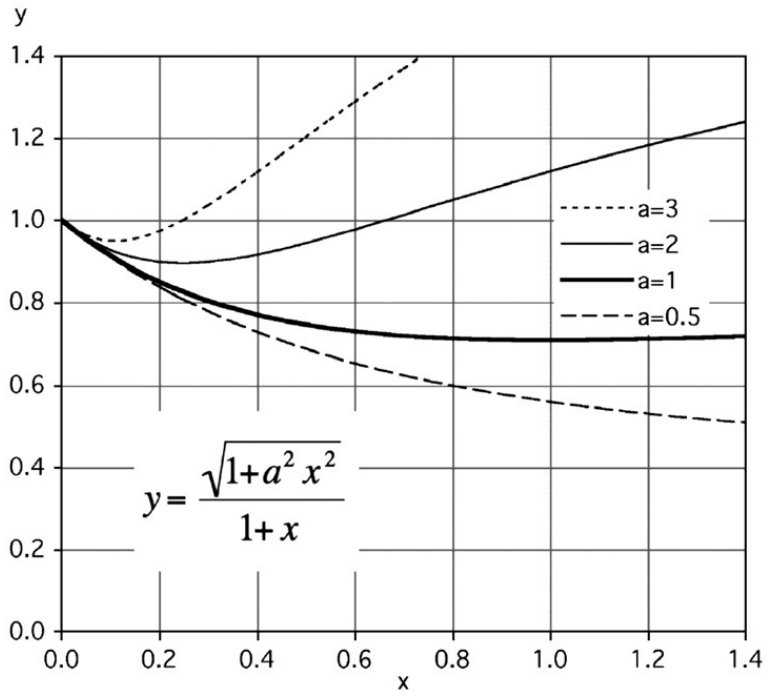

Fig. 5. Ratio of the relative errors of $w=v_1+v_2$ and v_1 (v_i having mean μ_i and standard deviation σ_i) plotted as function of $x=(\mu_2/\mu_1)$, with $a=(\sigma_2/\mu_2)/(\sigma_1/\mu_1)$.

estimate in this domain. Therefore one could consider only the uncertainty of mortality and take its σ_g as an appropriate estimate of the uncertainty of the total damage cost of these pollutants, but that is an upper limit.

For the sum of the cost of the classical air pollutants and the cost of greenhouse gases the relative errors are quite different, the geometric standard deviation being about 3 for the former and maybe around 6 for the latter. If the relative error of v_2 is a times that of v_1:

$$\frac{\sigma_2}{\mu_2} = a\frac{\sigma_1}{\mu_1},$$

then the relative error of the sum $w=v_1+v_2$ is

$$\frac{\sigma}{\mu} = \frac{\sigma_1}{\mu_1}\frac{\sqrt{1+a^2(\mu_2/\mu_1)^2}}{1+(\mu_2/\mu_1)}$$

and the ratio of the relative errors of w and v_1 is $y = \dfrac{\sigma/\mu}{\sigma_1/\mu} = \sqrt{\dfrac{1+a^2x^2}{1+x}}$.

It is plotted vs $x=(\mu_2/\mu_1)$ in Fig. 5.

If the geometric standard deviations are 3 for v_1 and 6 for v_2, Eqs. (16) and (17) imply that $a=3.2$. Then the geometric standard deviation σ_g of the sum is smaller than that of v_1, i.e. smaller than 3, as long as μ_2 is smaller than about 0.22 μ_1. In the limit where the total cost is dominated by v_2 (i.e. limit $x\to\infty$), σ_g of w approaches of course the one of v_2, 6 in this case. These examples illustrate a general rule: the absolute error of a sum is larger than the absolute errors of the summands. However, in many cases of interest for external costs the relative error of the sum is smaller than the relative errors of the summands.

More generally we have developed the following approximation for the important case of the sum of several lognormal variables. To begin we have verified with Monte Carlo calculations for a large number of realistic cases that the sum of lognormally distributed damage costs is, to an excellent approximation, also lognormal, as illustrated in Fig. 6 for some typical cases. Then we have derived two analytical estimates for the geometric standard deviation of the sum, one an over—the other an underestimation. Finally we have found that the average of these two estimates turns out to be remarkably close to the correct answer.

The first estimate is based on Eqs. (16) and (17) for the relation between $\{\mu, \sigma\}$ and $\{\mu_g, \sigma_g\}$ of the lognormal distribution. Using Eq. (16) one calculates the ordinary mean μ_j for each of the lognormal distributions j, and then one adds them according to the usual rule to obtain the mean of the sum

$$\mu_{est1} = \sum_j \mu_j = \sum_j \mu_{g,j}\exp\left(\frac{[\ln(\sigma_{g,j})]^2}{2}\right). \tag{24}$$

Using Eq. (17) with the usual rule for combining standard deviations one obtains the standard deviation of the sum

$$\sigma_{est1} = \sqrt{\sum_j \mu_{g,j}^2\left[\exp(2[\ln(\sigma_{g,j})]^2) - \exp\left([\ln(\sigma_{g,j})]^2\right)\right]}. \tag{25}$$

Now the first estimate of the geometric mean $\mu_{g,est1}$ and geometric standard deviation $\sigma_{g,est1}$ of the sum can be found by inverting Eqs. (16) and (17).

For the second estimate we calculate the quantities $\mu_{eq,j}$ and $\sigma_{eq,j}$ for the distribution of each term j in the sum according to the equations

$$\begin{aligned}\mu_{eq,j} &= 0.5\times[\mu_{g,j}\times\sigma_{g,j} + \mu_{g,j}/\sigma_{g,j}] \text{ and}\\ \sigma_{eq,j} &= 0.5\times[\mu_{g,j}\times\sigma_{g,j} - \mu_{g,j}/\sigma_{g,j}],\end{aligned} \tag{26}$$

i.e. we calculate an equivalent mean $\mu_{eq,j}$ as midpoint of the confidence interval of the lognormal distribution and an equivalent standard deviation $\sigma_{eq,j}$ as half width of this confidence interval. For $\mu_{eq,sum}$ and $\sigma_{eq,sum}$ of the sum we follow the usual rules, as we did for Eqs. (24) and (25),

$$\mu_{eq,sum} = \sum_j \mu_{eq,j} \text{ and } \sigma_{eq,sum} = \sqrt{\sum_j \sigma_{eq,j}^2}. \tag{27}$$

Then, we apply Eq. (26) to the sum (taking it with the subscript sum instead of j) and solve for $\mu_{g,est}$ and $\sigma_{g,est}$ of the sum to obtain the equations

$$\mu_{g,est2} = \mu_{eq,sum}\sqrt{1-\left(\frac{\sigma_{eq,sum}}{\mu_{eq,sum}}\right)^2} \quad \text{and} \quad \sigma_{g,est2} = \sqrt{\frac{1+\left(\frac{\sigma_{eq,sum}}{\mu_{eq,sum}}\right)}{1-\left(\frac{\sigma_{eq,sum}}{\mu_{eq,sum}}\right)}} \tag{28}$$

as an estimate of its geometric mean and geometric standard deviation.

We have compared these two approximations with the results of Monte Carlo calculations about 20 cases covering the ranges of the individual $\sigma_{g,j}$ that one is likely to encounter for air pollution damages, and we find we find that the first approximation overestimates σ_g of the sum by about 20% whereas the second

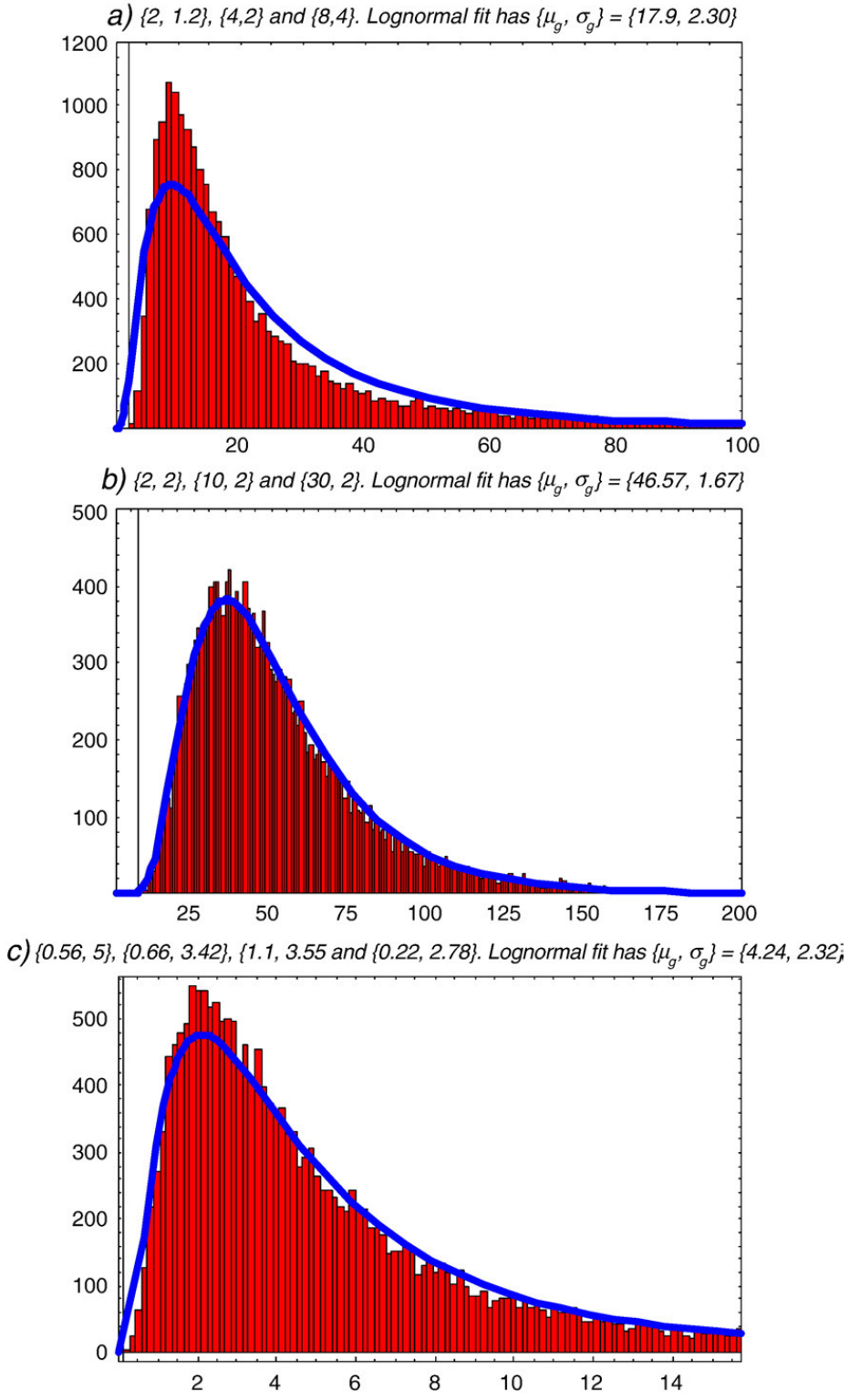

Fig. 6. Three examples of frequency distribution of the sum of lognormal variables, as well as fits by lognormal functions. Geometric mean and geometric standard deviation of the summands are indicated in the form $\{\mu_g, \sigma_g\}$.

underestimates it by a comparable amount. Taking the simple averages

$$\mu_{g,est} = (\mu_{g,est1} + \mu_{g,est2})/2 \text{ and}$$
$$\sigma_{g,est} = (\sigma_{g,est1} + \sigma_{g,est2})/2 \quad (29)$$

one comes very close to the exact result, as shown by the examples in Fig. 7. For example in part c) of Fig. 6 the four summands are chosen to correspond to the contributions of CO_2, SO_2, NO_2 and PM10 to the damage cost per kWh of electricity from coal in Europe, and the estimates

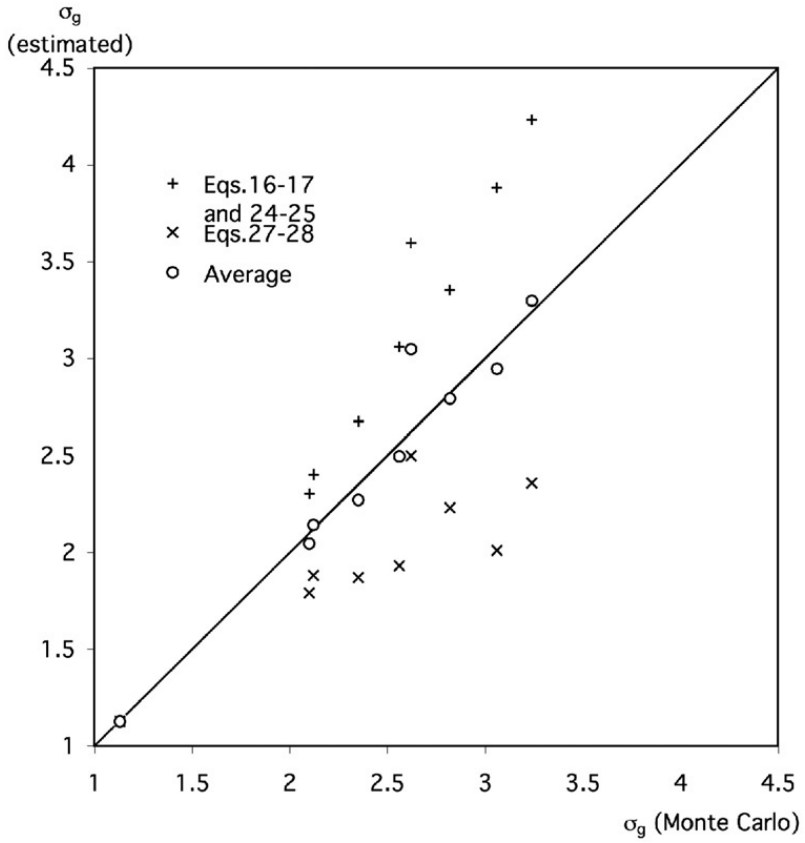

Fig. 7. Comparison between the geometric standard deviation estimated by Eqs. (16) and (17), by Eqs. (26)–(28) and by their average, and a Monte Carlo calculation, for the sum of three lognormal distributions.

of Eq. (29) are $\mu_{g,est}=(3.52+4.51)/2=4.02$ and $\sigma_{g,est}=(2.9+1.7)/2=2.3$, very close to the Monte Carlo result $\mu_g=4.24$ and $\sigma_g=2.32$.

5. Conclusion

Starting with the observation that the calculation of damage costs involves essentially products of statistically independent terms (representing exposure, dose-response function and monetary value), we have presented a simple and transparent estimation of uncertainties, as alternative to the Monte Carlo approach. It is appropriate for many applications, in particular air pollution damages. Depending on the details of the modeling of the environmental impact pathways, a more detailed uncertainty analysis may be desired for some of the terms in the product, in particular for the exposure. In such a case the simple method can also be combined with a more detailed Monte Carlo analysis of some elements of the impact pathway analysis.

As an example we have reported a Monte Carlo analysis of atmospheric dispersion. By estimating the uncertainties of the input parameters of the damage cost calculation for air pollutants we have determined the confidence intervals of the result for mortality, the end point that has been found to have the largest cost in all the recent assessments of air pollution (ORNL/RFF, 1994; Rowe et al., 1995; Abt, 2000, 2004; ExternE, 1998, 2000, 2004). Our key finding is that the uncertainty of pollution damage costs can be characterized, to good approximation, by lognormal distributions. They can be interpreted in terms of multiplicative confidence intervals of the lognormal distribution: if a cost has been estimated to be μ_g (geometric mean ≈ median) with geometric standard deviation σ_g, the probability is approximately 68% that the true value is in the interval $[\mu_g/\sigma_g, \mu_g.\sigma_g]$ and 95% that it is in $[\mu_g/\sigma_g^2, \mu_g.\sigma_g^2]$. Following the practice of the physical sciences we show error bars corresponding to 1 standard deviation, unlike epidemiology and the social sciences where 95% confidence intervals (≈2 standard deviations) are usually reported.

We have also provided rules for the placement of the confidence intervals in the typical case where the results are reported as means: the placement around the means is not symmetric on a logarithmic scale because of large differences between medians and means. Typically the geometric standard deviation is about 3 for most impacts

of air pollutants, in particular health impacts due to inhalation, but for global warming and for health impacts via ingestion the geometric standard deviation is larger, perhaps around 4 or 5 (McKone and Ryan, 1989; Spadaro and Rabl, 2004; Rabl et al., 2005).

We have also presented a simple method for estimating the uncertainty of the sum of damage costs for a set of different impact types or pollutants, and we have verified its usefulness by means of detailed Monte Carlo calculations. Whereas the absolute error of a sum is larger than the absolute errors of the summands, the relative error is smaller than the relative error of the summands: in the cases we have examined for air pollution, the relative error of the sum can be very significantly smaller.

Even though Monte Carlo calculations have become easy thanks to readily available software packages, the cost (not only price but above all learning time) can still be significant for many people. Also, the output of a Monte Carlo calculation is opaque: being purely numerical, it is difficult to see how the result would change if the input parameters are changed. By contrast, our analytic method is easy to apply in a simple spreadsheet, and the contribution of individual terms is explicitly visible. In view of the problems (in particular the unavoidable expert judgment) of any assessment of uncertainties in the environmental field we believe that our simple analytic method is as just about as reliable as a Monte Carlo calculation — and it is transparent.

Glossary and nomenclature

CRF	concentration-response function
ERF	exposure-response function
PM	particulate matter
UWM	uniform world model (Eq. (18))
c	concentration
C	cost
D	damage (impact) in physical units
E	exposure
I	impact rate
q	emission rate
s_{CR}	slope of CRF
v_{dep}	deposition (or depletion) velocity
μ	mean
μ_g	geometric mean
σ	standard deviation
σ_g	geometric standard deviation

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Corrigendum

Corrigendum to “Estimating the uncertainty of damage costs of pollution: A simple transparent method and typical results” [Environmental Impact Assessment Review, Volume 28, Issue 2, Pages 166–183]

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In Section 3.5 of this paper the equations for the relation between ordinary means and geometric means are correct, but the placement of the confidence intervals is not. Really the confidence intervals can be indicated for the ordinary mean μ or for the geometric mean μ_g of the damage cost estimates, depending on whether the damage cost is specified as ordinary mean or as geometric mean. If the ordinary mean is chosen, the corresponding 68% confidence interval extends from μ/σ_g to $\mu \cdot \sigma_g$, σ_g being the geometric standard deviation. If the geometric mean μ_g is chosen, the corresponding 68% confidence interval extends from μ_g/σ_g to $\mu_g \cdot \sigma_g$. Thus the last two lines of Table 3 and the lines “upper/mean” and “lower/mean” in Fig. 4 should not be used.